HYGROTHERMAL NUMERICAL SIMULATION MODEL FOR EARTH-TO-AIR HEAT EXCHANGERS: VALIDATION PROCESS AND AN EXAMPLE OF SIMULATION

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ABSTRACT
The earth-to-air heat exchanger is a device used for pre-heating and pre-cooling of fresh air. Paper describes main principles of the newly developed numerical hygro-thermal model for transient simulation of earth-to-air heat exchangers. Thorough validation against an analytical solution as well as against long-term monitoring of real scale earth-to-air heat exchanger is presented.

KEY-WORDS
Earth-to-air heat exchanger, passive cooling, numerical simulation, validation, low-energy house

INTRODUCTION

Using ground for heat dissipation is a traditional principle. Because of high soil thermal capacity the temperature in certain depth remains rather stable. The earth-to-air heat exchanger (EAHX) is a buried pipe in the ground through which the air is sucked by means of fan into a building. When the ground temperature is lower than the ambient temperature air is cooled down. When the ground temperature is higher than ambient temperature air is heated up.

The need of a flexible tool for the prediction of an earth-to-air heat exchanger performance and deeper understanding of complex processes taking place in the EAHX led us to develop hygro-thermal model of the EAHX. In parallel, the operation of a simple EAHX connected with a mechanical ventilation system equipped with heat recovery for a low-energy family house is monitored. Measured data have been used for another validation and testing of the theoretical model.

THEORETICAL ANALYSIS

Geometrical setup of the EAHX used in the model is depicted in Figure 1.

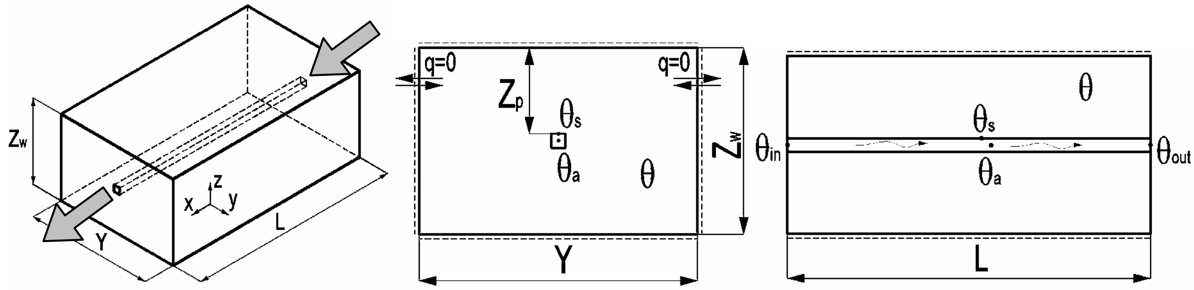

Figure 1: Axonometric view of the EAHX, longitudinal and transversal section of the EAHX

Heat transfer

The heat transfer processes in the earth-to-air heat exchanger are governed by three differential equations. The differential equation (1) describes a heat balance of air when latent heat generation is taken into account:

$$\frac{d\theta_a}{dx} + \frac{h_a 2\pi r_0}{m_a c_a}\theta_a - \frac{h_a 2\pi r_0}{m_a c_a}\theta_s - \frac{g_v l 2\pi r_0}{m_a c_a} = 0 \qquad (1)$$

where $\boldsymbol{\theta}_a$ is temperature of air in the pipe [°C], $\boldsymbol{\theta}_s$ is temperature of internal surface of the pipe [°C], $\boldsymbol{h}_a$ is convective air-to-pipe heat transfer coefficient [W/m^{2}K], $\boldsymbol{g}_v$ is condensing (+) or evaporating (-) amount of water vapor [kg/m^{2}s], $\boldsymbol{l}$ is latent heat of condensation [J/kg], $\boldsymbol{m}_a$ is air flow rate [kg/s], $\boldsymbol{c}_a$ is specific thermal capacity of air [J/kg.K], $\boldsymbol{r}_0$ is internal radius of pipe [m]. The differential equation (2) is the continuity (balance) equation describing heat conduction in the space around the pipe:

$$\rho c_p \frac{\partial \theta}{\partial t} = \lambda\left(\frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2}\right) \qquad (2)$$

where θ is temperature of the soil [°C], ρc_p volumetric heat capacity of the soil [J/m^3.K], λ is thermal conductivity of the soil [W/m.K]. The longitudinal component of the heat flow (along the length of the exchanger) was neglected. These two differential equations are linked to each other by the heat balance of the internal surface:

$$h_a(\theta_a - \theta_s) = -\lambda \left.\frac{\partial \theta}{\partial r}\right|_{si} \tag{3}$$

Moisture transfer

The description of moisture transfer in the EAHX is simplified. The moisture content variation and latent heat effects in the surrounding soil are not taken into account. It is assumed that air passing through a pipe, does not change its moisture content as long as no condensation or evaporation from the wet surface inside the pipe occurs:

$$\rho_v = \rho_{v,in} \tag{4}$$

where ρ_v is water vapor concentration of air in the EAHX [kg/m^3], $\rho_{v,in}$ is water vapor concentration of air at the inlet [kg/m^3]. The assumption is valid for a pipe made from plastic material with tight joints. If the water vapor condenses or evaporates, air is dehumidified or humidified. The moisture balance on the longitudinal control volume is set up analogously to the heat balance:

$$\frac{d\rho_v}{dx} + \frac{\beta_\rho 2\pi r_0}{V_a}\rho_v - \frac{\beta_\rho 2\pi r_0}{V_a}\rho_{v,sat}^{\theta_s} = 0 \tag{5}$$

where V_a is air flow rate [m^3/s], $\rho_{v,sat}$ is saturated water vapor concentration (a function of internal pipe surface temperature θ_s) a β_ρ is moisture transfer coefficient [m/s]. The moisture transfer coefficient can be calculated from the heat transfer coefficient according to Lewis formula:

$$\beta_\rho = \frac{h_a}{\rho_a \cdot c_a} \tag{6}$$ [1]

Furthermore, it is advantageous to set up these simplifications and limitations:

- moisture in the pipe can originate only as a consequence of previous condensation[2]
- originating moisture doesn't move in the pipe (moisture remains still in the same longitudinal control volume) and the surface of the pipe is moist uniformly[3]
- air moistening is delimited by saturated water vapor concentration

$$\rho_v \leq \rho_{v,sat}^{\theta_a} \tag{7}$$

where $\rho_{v,sat}$ is saturated water vapor concentration (a function of actual air temperature θ_a).

ALGORITHM

The following system of indexes is introduced: ***i*** - ***x*** axis index (along length of the exchanger), and ***t*** – time step index. Index (***t***-1) refers to previous values (preceding time step). Two basic situations can occur:

- operation - heat exchanger is in operation, $\mathbf{m}_a \neq 0$ and
- natural soil recovery[4] - heat exchanger is not in operation, $\boldsymbol{m}_a = 0$.

Stage of operation

For each time step these sub-steps are performed:

- Calculation of new air temperature: the analytical solution of (1) can be written as:

$$\theta_{a(i,t)} = \theta_{s,eqv_{(i,t-1)}} + \left(\theta_{a_{(i-1,t)}} - \theta_{s,eqv_{(i,t-1)}}\right)\exp^{-\frac{h_a 2\pi r_0}{m_a c_a}\Delta x_i} \tag{8}$$

$$\theta_{s,eqv\,(i,t-1)} = \theta_{s,mean\,(i,t-1)} + \frac{g_{v\,(i,t-1)} \cdot l}{h_a} \tag{9}$$

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[1] Adding formula (6) to (5) we get equation analogous to (1).

[2] In reality the important phenomenon can be the constant infiltration of moisture from the soil due to the leakage in the pipe wall as reported by [1].

[3] In reality the pipe is often sloped and the water can move.

[4] Forced soil recovery might occur during summer night when the exchanger is in operation (cooling of soil by relatively cold night air).

where $\theta_{s,mean}$ is mean surface temperature in the pipe [°C], $\theta_{s,eqv}$ is equivalent surface temperature (with influence of condensation or evaporation) [°C].

- Calculation of new water vapor concentration and moisture flux: the analytical solution of (5) can be written as:

$$\rho_{v(i,t)} = \rho_{v,sat_{(i,t-1)}} + \left(\rho_{v_{(i-1,t)}} - \rho_{v,sat_{(i,t-1)}}\right)\exp^{-\frac{\beta_\rho 2\pi r_0}{V_a}\Delta x_i} \tag{10}$$

The possibilities which can take place are summarized in the Figure 2.

<table>
<tr><th></th><th>Condensation</th><th colspan="2">Evaporation</th><th>Dry surface</th></tr>
<tr><td></td><td>$\rho_{v(i-1,t)} > \rho^{\theta_s}_{v,sat\,(i,t-1)}$</td><td colspan="2">$\rho_{v(i-1,t)} < \rho^{\theta_s}_{v,sat\,(i,t-1)}$ and $G_{acu\,(i,t-1)} > 0$ *</td><td>$\rho_{v(i-1,t)} < \rho^{\theta_s}_{v,sat\,(i,t-1)}$ and $G_{acu\,(i,t-1)} = 0$</td></tr>
<tr><td rowspan="3">Water vapor concentration</td><td rowspan="3">$\rho_{v(i,t)}$ = according to [10]</td><td colspan="2">$\rho_{v(i,t)}$ = according to [10]</td><td rowspan="3">$\rho_{v(i,t)} = \rho_{v(i-1,t)}$</td></tr>
<tr><td>$\rho_{v(i,t)} \le \rho^{\theta_a}_{v,sat}$</td><td>$\rho_{v(i,t)} = \rho_{v(i,t)}$</td></tr>
<tr><td>$\rho_{v(i,t)} > \rho^{\theta_a}_{v,sat}$</td><td>$\rho_{v(i,t)} = \rho^{\theta_a}_{v,sat}$ **</td></tr>
<tr><td rowspan="2">Condensing or evaporating amount</td><td rowspan="2">$g_{v(i,t)} = \dfrac{\left(\rho_{v(i-1,t)} - \rho_{v(i,t)}\right)V_a}{2\pi r_0 \Delta x_i}$</td><td colspan="2">$g_{v(i,t)} = \dfrac{\left(\rho_{v(i-1,t)} - \rho_{v(i,t)}\right)V_a}{2\pi r_0 \Delta x_i}$</td><td rowspan="2">$g_{v(i,t)} = 0$</td></tr>
<tr><td colspan="2">$g_{v(i,t)} = 0$ **</td></tr>
<tr><td colspan="5">*Inner surface of pipe is wet, G_{acu} is accumulated moisture within longitudinal control volume [kg].
**The relative humidity of air can't exceed 100%. If the air is fully saturated by water vapor the evaporation doesn't take place (although the pipe could be wet yet).</td></tr>
</table>

Figure 2: Condensation and evaporation in the model

- Calculation of new temperatures in the soil: the procedure uses already calculated air temperature with the convective resistance as air/pipe-to-soil boundary condition. Soil temperatures are calculated in certain number of perpendicular planes to the length of the exchanger (Figure 3). The explicit finite difference method (fully explicit scheme for discretization in time) is used for the solution of (2). The pipe is approximated by a square with thermal resistance and with perimeter which equals to perimeter of the pipe.
- Calculation of new pipe surface temperature: the surface temperature along the exchanger is calculated by setting up the heat balance for the inner surface of a fictitious pipe.

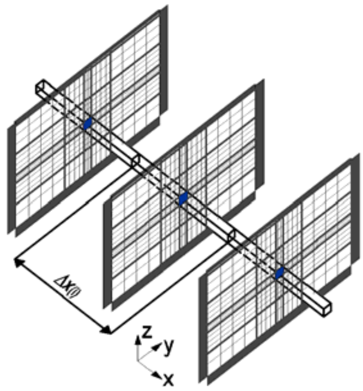


Figure 3: Perpendicular sections along length of the exchanger

Natural soil recovery

For this mode, air between panels is assumed to be still. Initial values of mass temperature are taken from the previous time step (the last time step of the operation). Again, explicit FDM is used for the calculation of the soil temperature field.

VALIDATION

The analytical solution for cylindrical heat exchanger (Figure 4) with external adiabatic boundary condition and harmonic oscillation at the input was used for comparison with numerical calculation.

The analytical solution [2] for harmonic input with amplitude θ_0 and angular frequency ω:

$$\theta_{in}(t) = \theta_0 \cos(\omega t) \tag{11}$$

is described by formula:

$$\theta_a(x,t) = \theta_0 \exp\left(-\frac{Sh}{m_a c_a}\right)\cos\left(\omega\left(t - \frac{x}{v_a}\right) - \frac{Sk}{m_a c_a}\right) \tag{12}$$

where S is heat exchange surface [m^2] from the inlet to distance x, h is total (air/pipe + soil) amplitude dampening exchange coefficient [W/m^2.K], k is total (air/pipe + soil) phase-shifting exchange coefficient

[W/m^2.K], $\boldsymbol{t}$ is time [s]. Term $\boldsymbol{x}/\boldsymbol{v}_\mathrm{a}$ is time for which air flows from the inlet to distance $\boldsymbol{x}$ from the inlet (transit time).

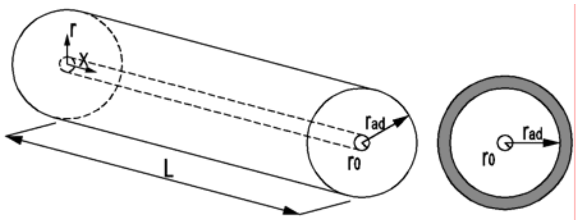


Figure 4: Cylindrical air-to-mass heat exchanger

The comparative analysis was made for three different setups of exchanger (Figure 5) which lead to different modulation of output temperature (annual dampening, daily dampening, and annual phase-shifting). These setups were used for validation of another numerical model [1].

Setup	$\boldsymbol{r}_0$ [m]	$\boldsymbol{r}_\mathrm{ad}$ [m]	$\boldsymbol{L}$ [m]	External boundary
Annual dampening	0.125	2.0	50	adiabatic
Daily dampening	0.125	0.6	50	adiabatic
Annual phase-shifting	0.125	0.6	400	adiabatic

Figure 5: Configurations of the cylindrical heat exchanger

The inlet air temperature was given by meteorological data for Geneva and was fully decomposed through Fourier series into a complete sum of harmonic pulses (4380 frequencies). Defined geometrical setups were submitted to constant air flow rate 162.5 m^3/h (approx. 200 kg/h) which induced the value 4.13 W/m^2.K of the convective heat transfer coefficient $\boldsymbol{h}_\mathrm{a}$. Soil properties were considered to be λ = 1.9 W/m.K, ρc_p = 1.9 MJ/m^3.K. The pipe was approximated by an equivalent square with perimeter which equals to perimeter of the pipe. The equivalent square for the external boundary was derived from the condition of identical surface areas.

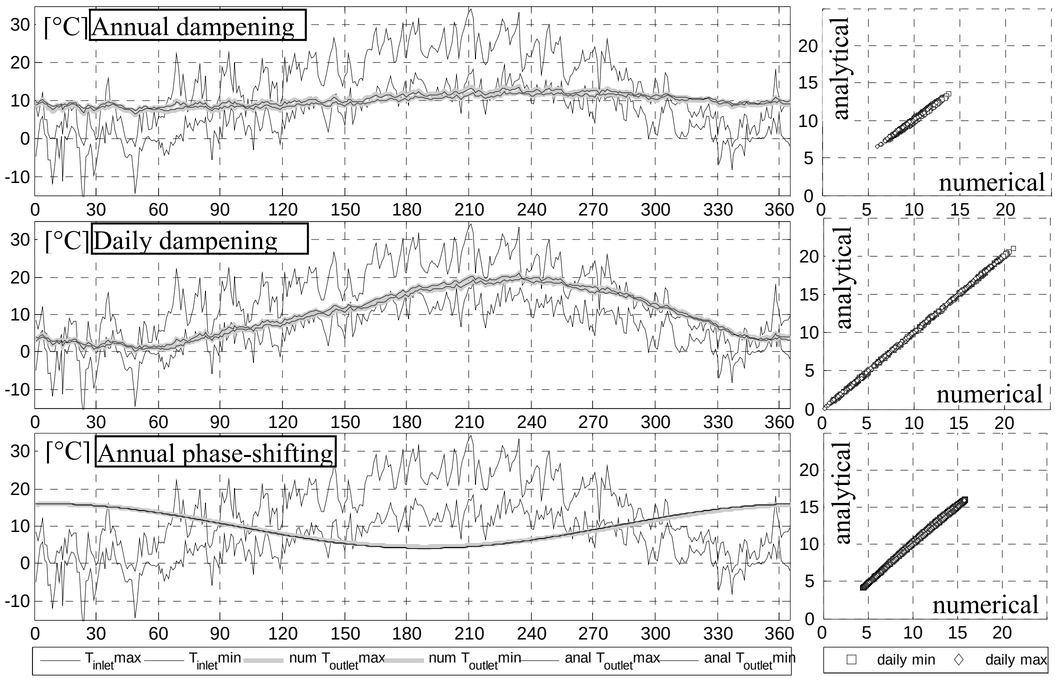


Figure 6: Left - comparison between numerical simulation (grey thick lines) and analytical solution (black thin lines) for different configurations of cylindrical heat exchanger with external adiabatic boundary conditions – daily values of maximal and minimal inlet and outlet temperatures; Right – scatter plot numerical vs. analytical (from daily maximal and minimal values)

The numerical vs. analytical analysis (Figure 6) showed perfect agreement of the numerical simulation with the analytical solution. Even if quite coarse mesh was used good accuracy was achieved.

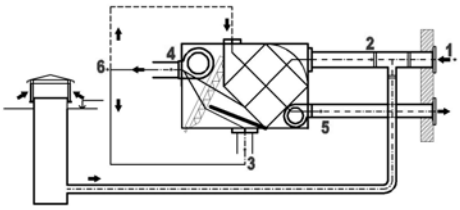
The third setup of the exchanger (400 m pipe length and 0.60 m soil radius) yields atypical performance. The outlet temperature is dampened similarly to daily dampening setup (50 m length and 0.60 m soil radius) but output signal is shifted much more. This shifting phenomenon has also daily mode [2].

A validation of moisture calculations appears to be another challenge. The analysis would need to measure inlet and outlet air relative humidity in order to determine moisture difference (condensation or evaporation) between inlet and outlet.

COMPARISON WITH MEASUREMENT IN-SITU

Measurement in-situ

The low-energy family house ventilated by mechanical ventilation equipped with heat recovery and simple earth-to-air heat exchanger has been monitored since the end of summer 2004. A scheme of monitored system with placement of sensors is in Figure 7. Figure 8 gives a basic description of the EAHX. The monitoring has imperfections. The interval of the measurement (20 min) is too long, air flow rate is not measured directly (it is estimated on the basis of actual ventilation regime) and the relative humidity of inlet and outlet air is not measured. The ventilation regime is often difficult to figure out. Despite these imperfections some sections of measurement can be used for a comparison with model prediction.



1. Temperature of ambient air
2. Outlet air temperature
3. Temperature of circulation air
4. Temperature (circulation + fresh air)
5. Temperature of waste air (after heat recovery)
6. Internal air temperature (in staircase space)

Figure 7: Scheme of monitored system with placement of sensors

Number of ducts	Length of pipe [m]	Diameter [mm]	Depth [m]
1	21	200	1,9
Air flow rate [m^3/(h.pipe)]	Soil	Control strategy	Place
100 – 350, higher values for summer ventilation	thermal diffusivity $a_s = 10^{-6}$ used in the simulation	According to θ_e and link to actual regime of ventilation unit	Velké Popovice

Figure 8: Description of evaluated EAHX

Simulation

The simulation was performed in time schedules according to Figure 9. For comparison with measured data these time sections were chosen: a) 25.2. – 06.3.2005 and b) 25.7. – 31.7.2005. The first section is the last term of air pre-heating during winter 04/05; the second section is the second term of air pre-cooling during summer 05, (see Figure 10 and Figure 11).

	initiation*	start 30.8.2004	→					finish 23.8.2005
0	0	0	0	0	0	0	0	0	0
A	0	125	125	125	0	350	0	350	0
B	0	350	350	350	350	350	350	350	350
		30.8. 31.12.	24.2.	6.3.		30.5.		31.7.	23.8.

	EAHX is ot in operation, soil temperature field is only influenced by ambient air temperature and incoming solar radiation on soil surface.
	Operation of the EAHX is intermittent; turned on when θ_{in} = 0°C, turned off when θ_{in} = +5°C. ** A number in the frame is air flow rate [m^3/h].
	Operation of the EAHX is intermittent. Identical operation as observed operation via monitoring.
	Operation of the EAHX is continuous.

Figure 9: Specification of the simulation

* Initiation is the simulation of a year according to schedule 0. The main purpose is to build up the undisturbed soil temperature field.

** In reality, EAHX operation is also linked to actual ventilation regime. The circulation of air with intermittent ventilation is the typical winter regime of ventilation unit.

The quality of the model prediction is dependent on accurrate estimation of natural thermal stratification in soil (initiation of simulation). The operation of EAHX disturbs this natural stratification.

For continuous mode the influence of operation on the soil temperature can be significant, especially during winter (see Figure 12, right). If the inlet air temperature is lower (or higher, it depends on season) than actual soil temperature the forced soil recovery takes place. Therefore the soil temperature remains oscillating around the undisturbed soil temperature during summer when forced soil recovery is more often. For intermittent mode the process of natural soil recovery is quite rapid, especially initially after turning the exchanger off (Figure 12, left).

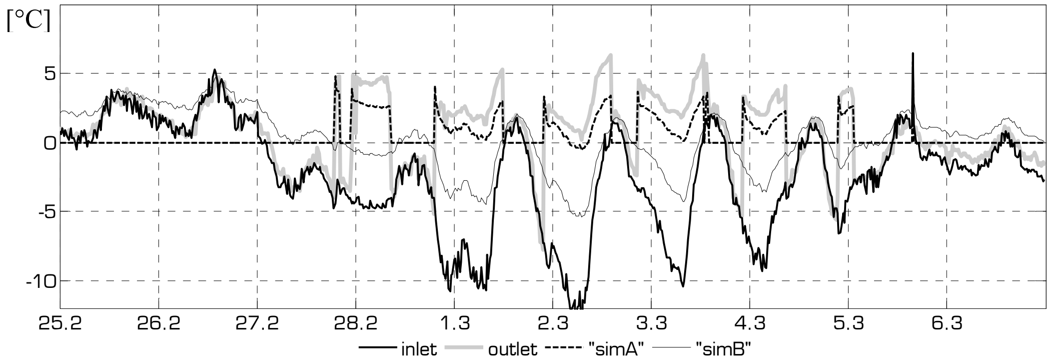


Figure 10: 25.02. – 06.03.2005; measured temperatures of inlet and outlet air vs. simulation

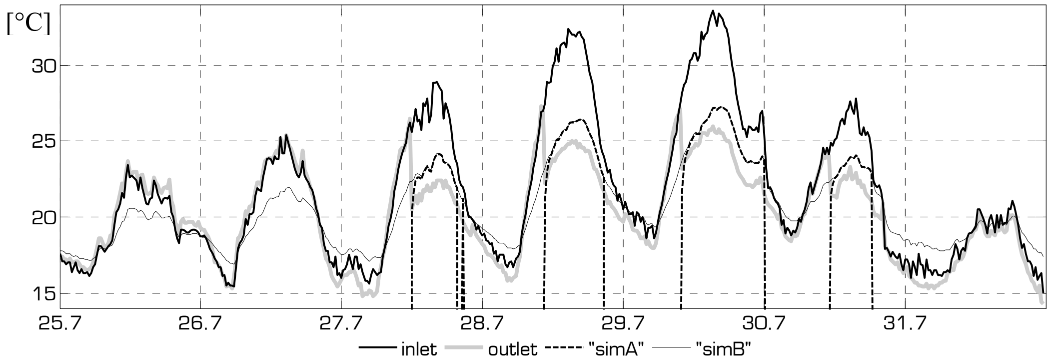


Figure 11: 25.7. – 31.7.2005; measured temperatures of inlet and outlet air vs. simulation

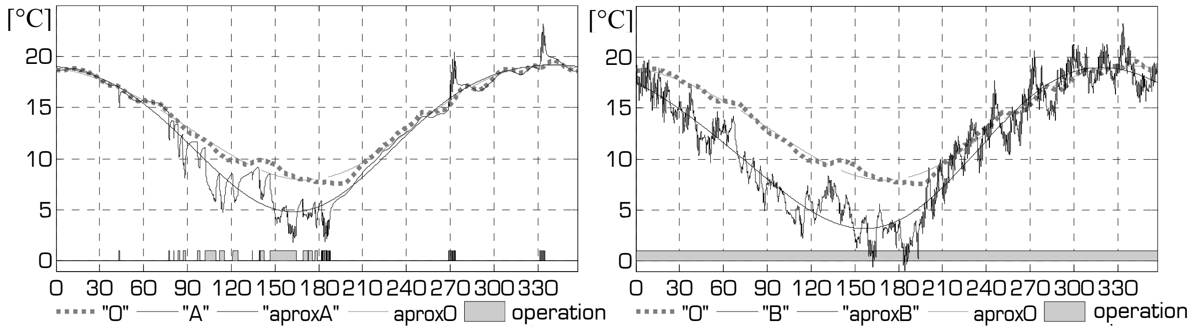


Figure 12: reference soil temperature during simulation (temperature in point 0.5 m far from pipe beginning in depth of 1.8 m)

CONCLUSION

The model brings clear information about processes which take place during earth-to-air heat exchanger operation. From this point of view the absolute fit between measured data and simulation is not so important.

The new measurement of another system was started. The measurement was designed with respect to data completeness and quality. We hope this measurement will help us in further model developing and with evaluation of not so common closed loop setup of the exchanger.

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